NAG Toolbox for MATLAB

f07ju

1 Purpose

f07ju computes the reciprocal condition number of a complex n by n Hermitian positive-definite tridiagonal matrix A, using the LDL^{H} factorization returned by f07jr.

2 Syntax

```
[rcond, info] = f07ju(d, e, anorm, 'n', n)
```

3 Description

f07ju should be preceded by a call to f07jr, which computes a modified Cholesky factorization of the matrix A as

$$A = LDL^{\mathrm{H}},$$

where L is a unit lower bidiagonal matrix and D is a diagonal matrix, with positive diagonal elements. f07ju then utilizes the factorization to compute $\left\|A^{-1}\right\|_1$ by a direct method, from which the reciprocal of the condition number of A, $1/\kappa(A)$ is computed as

$$1/\kappa_1(A) = 1/\left(\|A\|_1\left\|A^{-1}\right\|_1\right).$$

$1/\kappa(A)$ is returned, rather than $\kappa(A)$, since when A is singular $\kappa(A)$ is infinite.

4 References

Higham N J 2002 *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

5 Parameters

5.1 Compulsory Input Parameters

1: **d**$(*)$ **– double array**

Note: the dimension of the array **d** must be at least $\max(1, \mathbf{n})$.

Must contain the n diagonal elements of the diagonal matrix D from the LDL^{H} factorization of A.

2: **e**$(*)$ **– complex array**

Note: the dimension of the array **e** must be at least $\max(1, \mathbf{n} - 1)$.

Must contain the $(n - 1)$ subdiagonal elements of the unit lower bidiagonal matrix L. **e** can also be regarded as the superdiagonal of the unit upper bidiagonal matrix U from the $U^{\mathrm{H}}DU$ factorization of A.

3: **anorm – double scalar**

$\|A\|_1$. **anorm** may be computed by calling Missing 'id' with the argument **norm_p** = 'O', and must be computed either before calling f07jr, or else from a copy of the original matrix A.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The dimension of the array **d**.

n, the order of the matrix *A*.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

rwork

5.4 Output Parameters

1: **rcond – double scalar**

The reciprocal condition number, $1/\kappa_1(A) = 1/\left(\|A\|_1\|A^{-1}\|_1\right)$.

2: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows: 1: **n**, 2: **d**, 3: **e**, 4: **anorm**, 5: **rcond**, 6: **rwork**, 7: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed condition number will be the exact condition number for a closely neighbouring matrix.

8 Further Comments

The condition number estimation requires $O(n)$ floating-point operations.

See Section 15.6 of Higham 2002 for further details on computing the condition number of tridiagonal matrices.

The real analogue of this function is f07jg.

9 Example

```
d = [16;
     9;
     1;
     4];
e = [complex(1, +1);
     complex(2, -1);
     complex(1, -4)];
anorm = 83.75202879546762;
[rcond, info] = f07ju(d, e, anorm)
```

```
rcond =
   1.0862e-04
info =
           0
```